A Bridge Circuit Puzzle

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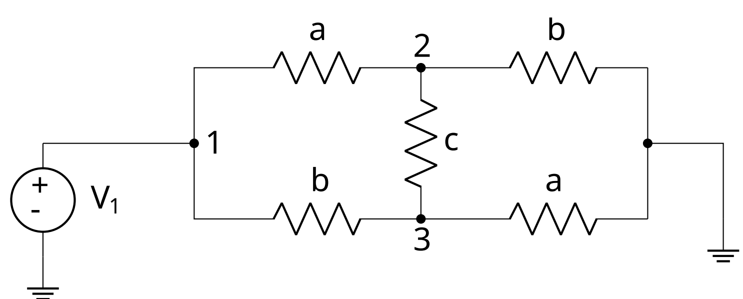
In the bridge circuit shown below, find the value of the resistance c such that the equivalent resistance connected across the voltage source is also equal to c. Show that for this value of c, the voltage across c is equal to $(\sqrt{b} - \sqrt{a})/(\sqrt{b} + \sqrt{a})$. Try to solve this problem yourself before looking at the solution below.



Start by finding the equivalent resistance. The easiest way to do this is to replace the voltage source with a current source I, and then find the voltage V_1 at node 1. The equivalent resistance is then V_1/I. This involves solving 3 equations for nodes 1, 2 and 3. The equations are Kirchoff's current law applied to the nodes. Leaving out the details, the equivalent resistance is

$$R = \frac{2ab + (a+b)c}{a+b+2c}$$

If you set $R = c$ and solve for c, you get $c = \sqrt{ab}$. In other words, when

c is equal to the geometric mean of a and b, the equivalent resistance is equal to c.

To solve the second part of the problem we need an expression for the voltage across c. Going back to the voltage source and writing the node equations for nodes 2 and 3, we get

$$(1/a + 1/b + 1/c)V_2 - (1/c)V_3 = (1/a)V_1$$

$$-(1/c)V_2 + (1/a + 1/b + 1/c)V_3 = (1/b)V_1$$

Solving these 2 equations for V_2/V_1 and V_3/V_1 we get the following expression for the voltage across resistor c.

$$\frac{V_2 - V_3}{V_1} = \frac{(b-a)c}{(b+a)c + 2ab}$$

If you substitute the value $c = \sqrt{ab}$ into this equation, you find that

$$\frac{V_2 - V_3}{V_1} = \frac{\sqrt{b} - \sqrt{a}}{\sqrt{b} + \sqrt{a}}$$

This simple circuit can calculate the ratio of the sum and difference of the square roots of 2 numbers.