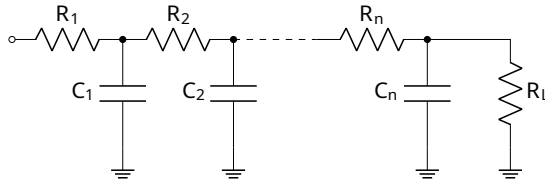


RC values for a Pink Noise Filter

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In our *Creating Noise* book we show how to use RC ladder networks to create pink noise. An n^{th} order network is shown below.



The R and C values are given by the following (equations 37 and 38 in the book).

$$C(n, i) = \frac{4i - 1}{(2(n - i) + 1)16^i} \frac{\binom{2(n+i)}{n+i}}{\binom{2(n-i)}{n-i}}$$
$$R(n, i) = \frac{(4i - 3)(2(n - i) + 1)16^i \binom{2(n-i)}{n-i}}{4(n - i + 1)(n + i)} \frac{\binom{2(n+i)}{n+i}}$$

Using these equations is a bit cumbersome so here are some asymptotic versions of the equations which will go into the next edition of the book.

$$C(n, i) = \frac{4i - 1}{(2(n - i) + 1)} \sqrt{\frac{n - i}{n + i}}$$

$$R(n, i) = \frac{(4i - 3)(2(n - i) + 1)}{4(n - i + 1)\sqrt{(n + i)(n - i)}}$$

$$C(n, i)R(n, i) = \frac{(4i - 1)(4i - 3)}{4(n - i + 1)(n + i)}$$

These equations obviously only work for $i < n$. For $i = n$ the asymptotic equations are

$$C(n, n) = \frac{4n - 1}{\sqrt{2\pi n}}$$

$$R(n, n) = \frac{(4n - 3)\sqrt{2\pi n}}{8n}$$

The last resistor in the chain has the asymptotic formula

$$R_L(n) = \sqrt{2\pi n}$$

In the limit $n \rightarrow \infty$ with $i/n \rightarrow x$ both the R and C equations have the following form

$$\frac{2x}{\sqrt{1 - x^2}}$$

where x has the range $0 < x < 1$. In this limit the ladder becomes a continuous transmission line and the total capacitance or resistance (measured from the beginning of the line) is found by integrating the above equation

$$C(x) = R(x) = 2 \left(1 - \sqrt{1 - x^2} \right)$$