Efficiently Heating a House III

Stefan Hollos
Exstrom Laboratories LLC
Longmont, Colorado
https://exstrom.com/
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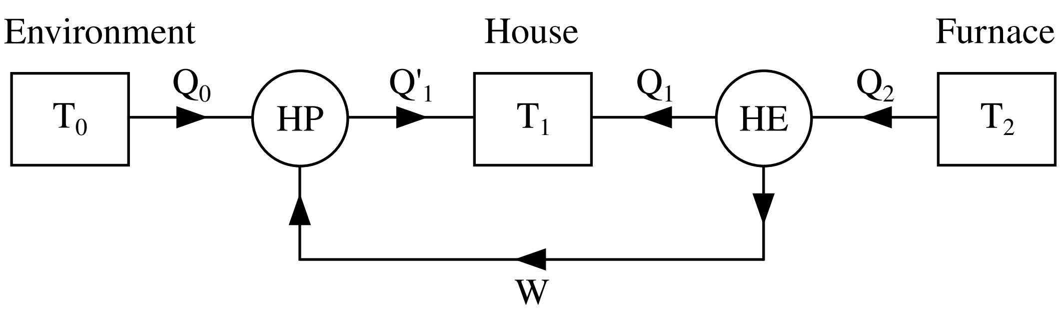
A heat pump needs a source of energy and most of the modern ones run on electricity. You just plug them into your house's electrical system and they'll start chugging away. Another interesting way to power a heat pump is with a heat engine. A thermodynamic diagram of this process is shown in the following figure.



There are three heat reservoirs, the outside environment at temperature T_0, the inside of the house at temperature T_1, and a high temperature heat source at T_2. The high temperature source could come from the combustion of some fuel such as natural gas. There is a heat engine operating between $T_2 \gg T_1$ that produces work W. This work is used to drive the heat pump that operates between $T_1 > T_0$ pumping heat into the house.

Does such a setup make sense? The answer is yes, at least from a theoretical perspective. It greatly improves on the efficiency of the heat pump alone, providing a much greater quantity of heat to the house for the same expenditure of energy. The method was first proposed by Lord Kelvin, one of the founders of thermodynamics, way back in 1853. Only a few thermodynamics books mention it, such as the one by Franzo H. Crawford published in 1963. A more modern reference is a paper by E. T. Jaynes published in 2003. Links to these references are given below.

So let's analyze this system to see how good it really is. The energy we have to pay for is Q_2 which comes from the combustion of fuel. The energy that we get going into the house is $Q_1 + Q_1'$ where Q_1 is the heat engine waste and Q_1' is the heat coming from the pump. So the efficiency, or heat gain, is the ratio of $Q_1 + Q_1'$ to Q_2. This is what we will calculate. The goal is to express the ratio in terms of the temperatures of the three reservoirs.

First we'll assume that both the pump and engine are reversible. This means they are as efficient as possible and produce no net entropy. Starting with the engine, we see from the diagram that the work it produces is given by (apply the first law of thermodynamics for a cyclic process)

$$W = Q_2 - Q_1$$

Since the engine produces no net entropy we must have (apply the second law of thermodynamics)

$$\frac{Q_2}{T_2} = \frac{Q_1}{T_1}$$

Using this equation, the work produced by the engine can be expressed as

$$W = Q_2 \left(1 - \frac{T_1}{T_2}\right)$$

This work is used by the pump to deliver heat Q_1' to the house so we have (again, apply the first law of thermodynamics for a cyclic process)

$$Q_1' = Q_0 + W$$

where Q_0 is the heat extracted from the outside environment. Since the pump produces no net entropy we have (apply the second law of thermodynamics)

$$\frac{Q_0}{T_0} = \frac{Q_1'}{T_1}$$

With this equation we can write the heat delivered to the house as

$$Q_1' = Q_1' \frac{T_0}{T_1} + W$$

Using the above expression for W and solving for Q_1' we get

$$Q_1' = Q_2 \frac{T_1}{T_2} \frac{T_2 - T_1}{T_1 - T_0}$$

The total heat delivered to the house is then

$$Q_H = Q_1' + Q_1 = Q_2 \frac{T_1}{T_2} \frac{T_2 - T_0}{T_1 - T_0}$$

so the heat gain is

$$G(T_0, T_1, T_2) = \frac{Q_H}{Q_2} = \frac{T_1}{T_2} \frac{T_2 - T_0}{T_1 - T_0}$$

which can also be written as

$$G(T_0, T_1, T_2) = \frac{1 - T_0/T_2}{1 - T_0/T_1}$$

As an example, assume the fuel we are using is methane, which has a combustion temperature in air of 1950°C = 2223K. Assume the outside temperature is 0°C = 273K, and assume the inside temperature is 20°C = 293K. Plugging these values into the heat gain formula we get an incredible $G(273, 293, 2223) = 12.85$. So for each unit of heat purchased this setup produces almost 13 units of heat in the house. It is important to remember however that we are assuming an ideal heat engine and pump that produce no entropy. With a real engine and pump we would probably only get about half this value.

We can analyze the system without the assumption of a ideal engine and pump by focusing on the overall change in entropy. Note, first of all, that the net result of a cycle is that Q_2 leaves the T_2 reservoir, and Q_0 leaves the T_0 reservoir, so the heat entering the house is $Q_H = Q_2 + Q_0$. The total entropy change of the system is

$$\Delta S = \frac{Q_H}{T_1} - \frac{Q_0}{T_0} - \frac{Q_2}{T_2}$$

In general, this must be greater than or equal to zero (it is only equal when the engine and pump are reversible). Therefore, we have

$$\frac{Q_H}{T_1} \geq \frac{Q_0}{T_0} + \frac{Q_2}{T_2} = \frac{Q_H - Q_2}{T_0} + \frac{Q_2}{T_2}$$

Rearranging this expression gives us

$$Q_H \leq Q_2 \frac{T_1}{T_2} \frac{T_2 - T_0}{T_1 - T_0}$$

When engine and pump are reversible, this becomes an equality and we get the previous result. The general expression for the heat gain is then

$$G(T_0, T_1, T_2) \leq \frac{1 - T_0/T_2}{1 - T_0/T_1}$$

This shows that the result we calculated earlier is the maximum possible heat gain, for real engines and pumps it will be smaller.

How could you actually realize such a system? One way is to use a natural gas powered electric generator to power an electric heat pump. [Electric generators](https://www.generac.com/for-homeowners/home-backup-power) are generally very efficient and you can also get very efficient electric heat pumps. You won't get anywhere near the ideal heat gain we calculated above, but it should be better than running just a heat pump alone off the electric grid. An additional benefit is that you have an electric backup generator in case the power ever goes out.

References

Sir William Thomson, Baron Kelvin, "Power required for the thermodynamic heating of buildings", Cambridge and Dublin Mathematical Journal, November, 1853. A copy of this paper is found in Kelvin's Volume V of Mathematical and Physical Papers, p124-133.

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