Combinatorics and Super Bowl LVIII

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Subjects: math, combinatorics
Mar 3, 2024

I don't watch a lot of football but this year I did watch Super Bowl LVIII between the San Francisco 49ers and the Kansas City Chiefs played February 11 2024 in Las Vegas NV. I think it may be the best football game I've ever seen. The final score was Kansas City 25, San Francisco 22 in overtime.

If you didn't have the chance to watch the game is there anything the score by itself can tell you about how the game was played? In American football, a team can score in the following ways:

- 6 points is a touchdown
- 7 points is a touchdown with 1 point conversion
- 8 points is a touchdown with 2 point conversion
- 3 points is a field goal
- 2 points is a safety

Touchdowns with 2 point conversions and safeties are rare. If we exclude those then how many ways can a team end up with 25 or 22 points? This amounts to finding all the integer partitions of 25 and 22 using the parts (3,6,7). For 25 there are 4 possible partitions: 3+3+3+3+3+3+7, 3+3+3+3+6+7, 3+3+6+6+7, 6+6+6+7. For 22 there are 3 possible partitions: 3+3+3+3+3+7, 3+3+3+6+7, 3+6+6+7.

If we're not interested in the order that the points were scored then the number of ways the final score can be 25 to 22 is number of partitions of 25 times the number of partitions of 22 or 4*3=12. If we are interested in the order of the points then we have to look at how many ways each partition could have occurred.

For the partitions of 25 we have:

- multiplicity(3+3+3+3+3+3+7)=7!/6!=7
- multiplicity(3+3+3+3+6+7)=6!/4!=30
- multiplicity(3+3+6+6+7)=5!/(2! 2!)=30
- multiplicity(6+6+6+7)=4!/3!=4

For the partitions of 22 we have:

- multiplicity(3+3+3+3+3+7)=6!/5!=6
- multiplicity(3+3+3+6+7)=5!/3!=20
- multiplicity(3+6+6+7)=4!/2!=12

The total number of ways the points could have occurred is then

(7 + 30 + 30 + 4) * (6 + 20 + 12) = 71*38 = 2698

The actual way the points were scored, including the order is

- Kansas City: (3,3,7,3,3,6)
- San Francisco: (3,7,6,3,3)

It is interesting that both of these ways come from partitions with high multiplicities. You could define a probability distribution over the partitions based on their multiplicities. For the partitions of 25 the probabilities are:

- probability(3+3+3+3+3+3+7)=7/71=0.098
- probability(3+3+3+3+6+7)=30/71=0.423

- probability(3+3+6+6+7)=30/71=0.423
- probability(6+6+6+7)=4/71=0.056

For the partitions of 22 the probabilities are:

- probability(3+3+3+3+3+7)=6/38=3/19=0.158
- probability(3+3+3+6+7)=20/38=10/19=0.526
- probability(3+6+6+7)=12/38=6/19=0.316

The Kansas City score came from a partition group with probability 0.423 and the San Francisco score came from a partition group with probability 0.526.

This kind of analysis is possible because of the different ways a team can score in American football. The score contains more information than it does in a sport like soccer where the score is only accumulated in one point increments. With more information you can narrow down the number of games even more. Just knowing that the game ended in overtime and knowing the rules of overtime play should narrow it down. Better yet would be knowing the score at the end of each quarter.